Net present value

In finance, the **net present value** (**NPV**) or **net present worth** (**NPW**)[1] is defined as the sum of the present values (PVs) of incoming and outgoing cash flows over a period of time. Incoming and outgoing cash flows can also be described as benefit and cost cash flows, respectively.[2]

Time value of money dictates that time has an impact on the value of cash flows. Cash flows of *nominal* equal value over a time series result in different *effective* value cash flows that makes future cash flows less valuable over time. If for example there exists a time series of identical cash flows, the cash flow in the present is the most valuable, with each future cash flow becoming less valuable than the previous cash flow. Thus, a cash flow today is more valuable than an identical cash flow in the future.[3] This decrease occurs because the discount factor represents the expected rate of return of each cash flow in a different investment with identical risk. With each additional period, the present value of a subsequent future cash flow decreases.

The NPV of an investment is determined by calculating the present value (PV) of the total benefits and costs which is achieved by discounting the future value of each cash flow (see Formula). NPV is a useful tool to determine whether a project or investment will result in a net profit or a loss because of its simplicity. A positive NPV results in profit, while a negative NPV results in a loss. The NPV measures the excess or shortfall of cash flows, in present value terms, above the cost of funds.[4] In a theoretical situation of unlimited capital budgeting a company should pursue every investment with a positive NPV. However, in practical terms a company's capital constraints limit investments to projects with the highest NPV whose cost cash flows do not exceed the company's capital. NPV is a central tool in discounted cash flow (DCF) analysis and is a standard method for using the time value of money to appraise long-term projects. It is widely used throughout economics, finance, and accounting.

In the case when all future cash flows are incoming (such as the principal and coupon payment of a bond) the only outflow of cash is the purchase price, the NPV is simply the PV of future cash flows minus the purchase price (which is its own PV). NPV can be described as the "difference amount" between the sums of discounted cash inflows and cash outflows. It compares the present value of money today to the present value of money in the future, taking inflation and returns into account.

The NPV of a sequence of cash flows takes as input the cash flows and a discount rate or discount curve and outputs a price. The converse process in DCF analysis — taking a sequence of cash flows and a price as input and inferring as output a discount rate (the discount rate which would yield the given price as NPV) — is called the yield and is more widely used in bond trading.

1 Formula

Each cash inflow/outflow is discounted back to its present value (PV). Then they are summed. Therefore NPV is the sum of all terms,

$$\frac{R_t}{(1+i)^t}$$

where

- t – the time of the cash flow
- i – the discount rate (the rate of return that could be earned on an investment in the financial markets with similar risk.); the opportunity cost of capital
- R_t – the net cash flow i.e. cash inflow – cash outflow, at time t. For educational purposes, R_0 is commonly placed to the left of the sum to emphasize its role as (minus) the investment.

The result of this formula is multiplied with the Annual Net cash in-flows and reduced by Initial Cash outlay the present value but in cases where the cash flows are not equal in amount, then the previous formula will be used to determine the present value of each cash flow separately. Any cash flow within 12 months will not be discounted for NPV purpose, nevertheless the usual initial investments during the first year R_0 are summed up a negative cash flow.[5]

Given the (period, cash flow) pairs (t , R_t) where N is the total number of periods, the net present value NPV is given by:

$$\mathrm{NPV}(i,N)=\sum_{t=0}^{N}\frac{R_t}{(1+i)^t}$$

Many computer-based spreadsheet programs have built-in formulae for PV and NPV.

2 The discount rate

Main article: Annual effective discount rate

The rate used to discount future cash flows to the present value is a key variable of this process.

A firm's weighted average cost of capital (after tax) is often used, but many people believe that it is appropriate to use higher discount rates to adjust for risk, opportunity cost, or other factors. A variable discount rate with higher rates applied to cash flows occurring further along the time span might be used to reflect the yield curve premium for long-term debt.

Another approach to choosing the discount rate factor is to decide the rate which the capital needed for the project could return if invested in an alternative venture. If, for example, the capital required for Project A can earn 5% elsewhere, use this discount rate in the NPV calculation to allow a direct comparison to be made between Project A and the alternative. Related to this concept is to use the firm's reinvestment rate. Reinvestment rate can be defined as the rate of return for the firm's investments on average. When analyzing projects in a capital constrained environment, it may be appropriate to use the reinvestment rate rather than the firm's weighted average cost of capital as the discount factor. It reflects opportunity cost of investment, rather than the possibly lower cost of capital.

An NPV calculated using variable discount rates (if they are known for the duration of the investment) may better reflect the situation than one calculated from a constant discount rate for the entire investment duration. Refer to the tutorial article written by Samuel Baker[6] for more detailed relationship between the NPV value and the discount rate.

For some professional investors, their investment funds are committed to target a specified rate of return. In such cases, that rate of return should be selected as the discount rate for the NPV calculation. In this way, a direct comparison can be made between the profitability of the project and the desired rate of return.

To some extent, the selection of the discount rate is dependent on the use to which it will be put. If the intent is simply to determine whether a project will add value to the company, using the firm's weighted average cost of capital may be appropriate. If trying to decide between alternative investments in order to maximize the value of the firm, the corporate reinvestment rate would probably be a better choice.

Using variable rates over time, or discounting "guaranteed" cash flows differently from "at risk" cash flows, may be a superior methodology but is seldom used in practice. Using the discount rate to adjust for risk is often difficult to do in practice (especially internationally) and is difficult to do well. An alternative to using discount factor to adjust for risk is to explicitly correct the cash flows for the risk elements using rNPV or a similar method, then discount at the firm's rate.

3 Use in decision making

NPV is an indicator of how much value an investment or project adds to the firm. With a particular project, if R_t is a positive value, the project is in the status of positive cash inflow in the time of *t*. If R_t is a negative value, the project is in the status of discounted cash outflow in the time of *t*. Appropriately risked projects with a positive NPV could be accepted. This does not necessarily mean that they should be undertaken since NPV at the cost of capital may not account for opportunity cost, *i.e.,* comparison with other available investments. In financial theory, if there is a choice between two mutually exclusive alternatives, the one yielding the higher NPV should be selected.

4 Interpretation as integral transform

The time-discrete formula of the net present value

$$\mathrm{NPV}(i) = \sum_{t=0}^{N} \frac{R_t}{(1+i)^t}$$

can also be written in a continuous variation

$$\mathrm{NPV}(i) = \int_{t=0}^{\infty} (1+i)^{-t} \cdot r(t)\,dt$$

where

> $r(t)$ is the rate of flowing cash given in money per time, and $r(t) = 0$ when the investment is over.

Net present value can be regarded as Laplace-[7][8] respectively Z-transformed cash flow with the integral operator including the complex number *s* which resembles to the interest rate *i* from the real number space or more precisely $s = \ln(1+i)$.

$$F(s) = \{\mathcal{L}f\}(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

From this follow simplifications known from cybernetics, control theory and system dynamics. Imaginary parts of the complex number *s* describe the oscillating behaviour (compare with the pork cycle, cobweb theorem, and phase shift between commodity price and supply offer) whereas real parts are responsible for representing the effect of compound interest (compare with damping).

5 Example

A corporation must decide whether to introduce a new product line. The company will have immediate costs of 100,000 at *t = 0*. Recall, a cost is a negative or outgoing cash flow, thus this cash flow is represented as $-100{,}000$. The company assumes the product will provide equal benefits of 10,000 for each of 12 years beginning at *t = 1*. For simplicity, assume the company will have no outgoing cash flows after the initial 100,000 cost. This also makes the simplifying assumption that the net cash received or paid is lumped into a single transaction occurring *on the last day* of each year. At the end of the 12 years the product no longer provides any cash flow and is discontinued without any additional costs. Assume that the effective annual discount rate is 10%.

The present value (value at *t = 0*) can be calculated for each year:

The total present value of the incoming cash flows is 68,136.92. The total present value of the outgoing cash flows is simply the 100,000 at time *t = 0*. Thus:

NPV $= PV(Benefits) - PV(Costs)$

Rearranging the formula:

NPV $= -PV(Costs) + PV(Benefits)$

In the above example:

NPV $= -100,000 + 68,136.92$

NPV $= -31,863.08$

Observe that as *t* increases the present value of each cash flow at *t* decreases. For example, the final incoming cash flow has a future value of 10,000 at *t = 12* but has a present value (at *t = 0*) of 3,186.31. The opposite of discounting is compounding. Taking the example in reverse, it is the equivalent of investing 3,186.31 at *t = 0* (the present value) at an interest rate of 10% compounded for 12 years, which results in a cash flow of 10,000 at *t = 12* (the future value).

The importance of NPV becomes clear in this instance. Although the incoming cash flows (10,000 x 12 = 120,000) appear to exceed the outgoing cash flow (100,000), the future cash flows are not adjusted using the discount rate. Thus, the project appears misleadingly profitable. When the cash flows are discounted however, it indicates the project would result in a net loss of 31,863.08. Thus, the NPV calculation indicates that this project should be disregarded because investing in this project is the equivalent of a loss of 31,863.08 at *t = 0*. The concept of time value of money indicates that cash flows in different periods of time cannot be accurately compared unless they have been adjusted to reflect their value at the same period of time (in this instance, *t = 0*).[3] It is the present value of each future cash flow that must be determined in order to provide any meaningful comparison between cash flows at different periods of time. There are a few inherent assumptions in this type of discounted cash flow / net present value type analysis:

1. The *investment horizon* of all possible investment projects considered are equally acceptable to the investor (e.g. a 3-year project is not necessarily preferable vs. a 20-year project.)
2. The 10% discount rate is the appropriate (and stable) rate to discount the expected cash flows from each project being considered. Each project is assumed equally speculative.
3. The shareholders cannot get above a 10% return on their money if they were to directly assume an equivalent level of risk. (If the investor could do better elsewhere, no projects should be undertaken by the firm, and the excess capital should be turned over to the shareholder through dividends and stock repurchases.)

More realistic problems would also need to consider other factors, generally including: smaller time buckets, the calculation of taxes (including the cash flow timing), inflation, currency exchange fluctuations, hedged or unhedged commodity costs, risks of technical obsolescence, potential future competitive factors, uneven or unpredictable cash flows, and a more realistic salvage value assumption, as well as many others.

A more simple example of the net present value of incoming cash flow over a set period of time, would be winning a Powerball lottery of $500 million. If one does not select the “CASH” option they will be paid $25,000,000 dollars per year for 20 years, a total of $500,000,000, however, if one does select the “CASH” option, they will receive a one-time lump sum payment of approximately $285 million, the NPV of $500,000,000 paid over time. See “other factors” above that could affect the payment amount. Both scenario’s are before taxes.

6 Common pitfalls

- If, for example, the *Rt* are generally negative late in the project (*e.g.*, an industrial or mining project might have clean-up and restoration costs), then at that stage the company owes money, so a high discount rate is not cautious but too optimistic. Some people see this as a problem with NPV. A way to avoid this problem is to include explicit provision for financing any losses after the initial investment, that is, explicitly calculate the cost of financing such losses.
- Another common pitfall is to adjust for risk by adding a premium to the discount rate. Whilst a bank might charge a higher rate of interest for a risky project, that does not mean that this is a valid approach to adjusting a net present value for risk, although it can be a reasonable approximation in some

specific cases. One reason such an approach may not work well can be seen from the following: if some risk is incurred resulting in some losses, then a discount rate in the NPV will reduce the impact of such losses below their true financial cost. A rigorous approach to risk requires identifying and valuing risks explicitly, *e.g.*, by actuarial or Monte Carlo techniques, and explicitly calculating the cost of financing any losses incurred.

- Yet another issue can result from the compounding of the risk premium. R is a composite of the risk free rate and the risk premium. As a result, future cash flows are discounted by both the risk-free rate as well as the risk premium and this effect is compounded by each subsequent cash flow. This compounding results in a much lower NPV than might be otherwise calculated. The certainty equivalent model can be used to account for the risk premium without compounding its effect on present value.
- Another issue with relying on NPV is that it does not provide an overall picture of the gain or loss of executing a certain project. To see a percentage gain relative to the investments for the project, usually, Internal rate of return or other efficiency measures are used as a complement to NPV.
- Non-specialist users frequently make the error of computing NPV based on cash flows after interest. This is wrong because it double counts the time value of money. Free cash flow should be used as the basis for NPV computations.

7 History

Net present value as a valuation methodology dates at least to the 19th century. Karl Marx refers to NPV as fictitious capital, and the calculation as "capitalising," writing:[9]

In mainstream neo-classical economics, NPV was formalized and popularized by Irving Fisher, in his 1907 *The Rate of Interest* and became included in textbooks from the 1950s onwards, starting in finance texts.[10][11]

8 Alternative capital budgeting methods

- Adjusted present value (APV): adjusted present value, is the net present value of a project if financed solely by ownership equity plus the present value of all the benefits of financing.
- Accounting rate of return (ARR): a ratio similar to IRR and MIRR
- Cost-benefit analysis: which includes issues other than cash, such as time savings.
- Internal rate of return (IRR): which calculates the rate of return of a project while disregarding the absolute amount of money to be gained.
- Modified internal rate of return (MIRR): similar to IRR, but it makes explicit assumptions about the reinvestment of the cash flows. Sometimes it is called Growth Rate of Return.
- Payback period: which measures the time required for the cash inflows to equal the original outlay. It measures risk, not return.
- Real option: which attempts to value managerial flexibility that is assumed away in NPV.
- Equivalent annual cost (EAC): a capital budgeting technique that is useful in comparing two or more projects with different lifespans.

9 See also

- Profitability index
- Internal Rate of Return

10 References

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[2] Berk, Johnathan; DeMarzo, Peter; Stangeland, David (2015). *Corporate Finance* (3rd Canadian Edition ed.). Toronto: Pearson Canada. p. 64. ISBN 978-0133552683.

[3] Berk, DeMarzo, and Stangeland, p. 94.

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[5] Khan, M.Y. (1993). *Theory & Problems in Financial Management*. Boston: McGraw Hill Higher Education. ISBN 978-0-07-463683-1.

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[7] Grubbström, Robert W. (1967). "On the Application of the Laplace Transform to Certain Economic Problems". *Management Science* **13**: 558–567. doi:10.1287/mnsc.13.7.558.

[8] Steven Buser: LaPlace Transforms as Present Value Rules: A Note, *The Journal of Finance*, Vol. 41, No. 1, March, 1986, pp. 243-247.

[9] Karl Marx, *Capital,* Volume 3, 1909 edition, p. 548

[10] Bichler, Shimshon; Nitzan, Jonathan (July 2010), *Systemic Fear, Modern Finance and the Future of Capitalism*, Jerusalem and Montreal, pp. 8–11 (for discussion of history of use of NPV as "capitalisation")

[11] Nitzan, Jonathan; Bichler, Shimshon (2009), *Capital as Power. A Study of Order and Creorder.*, RIPE Series in Global Political Economy, New York and London: Routledge

11 External links

- Download of spreadsheets for NPV calculation http://www.thebusinesscase.info (Download/For Professionals and Students/Fig. 2.6)

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