Time value of money

A **time value of money** calculation is one which solves for one of several variables in a financial problem. In a typical case, the variables might be: a balance (the real or nominal value of a debt or a financial asset in terms of monetary units); a periodic rate of interest; the number of periods; and a series of cash flows (in the case of a debt, these are payments against principal and interest; in the case of a financial asset, these are contributions to or withdrawals from the balance). More generally, the cash flows may not be periodic but may be specified individually. Any of the variables may be the independent variable (the sought-for answer) in a given problem. For example, one may know that: the interest is 0.5% per period (per month, say); the number of periods is 60 (months); the initial balance (of the debt, in this case) is 25,000 units; and the final balance is 0 units. The unknown variable may be the monthly payment that the borrower will need to pay.

For example, £100 invested for one year, earning 5% interest, will be worth £105 after one year; therefore, £100 paid now *and* £105 paid exactly one year later *both* have the same value to a recipient who expects 5% interest. That is, £100 invested for one year at 5% interest has a *future value* of £105.[1] This notion dates back at least to Martín de Azpilcueta (1491–1586) of the School of Salamanca.

This principle allows for the valuation of a likely stream of income in the future, in such a way that annual incomes are discounted and then added together, thus providing a lump-sum "present value" of the entire income stream; all of the standard calculations for time value of money derive from the most basic algebraic expression for the present value of a future sum, "discounted" to the present by an amount equal to the time value of money. For example, the future value sum FV to be received in one year is discounted at the rate of interest r to give the present value sum PV :

$$PV = \frac{FV}{(1+r)}$$

Some standard calculations based on the time value of money are:

- **Present value**: The current worth of a future sum of money or stream of cash flows, given a specified rate of return. Future cash flows are "discounted" at the *discount rate;* the higher the discount rate, the lower the present value of the future cash flows. Determining the appropriate discount rate is the key to valuing future cash flows properly, whether they be earnings or obligations.[2]

- **Present value of an annuity**: An annuity is a series of equal payments or receipts that occur at evenly spaced intervals. Leases and rental payments are examples. The payments or receipts occur at the end of each period for an ordinary annuity while they occur at the beginning of each period for an annuity due.[3]

  Present value of a perpetuity is an infinite and constant stream of identical cash flows.[4]

- **Future value**: The value of an asset or cash at a specified date in the future, based on the value of that asset in the present.[5]

- **Future value of an annuity (FVA)**: The future value of a stream of payments (annuity), assuming the payments are invested at a given rate of interest.

1 Calculations

There are several basic equations that represent the equalities listed above. The solutions may be found using (in most cases) the formulas, a financial calculator or a spreadsheet. The formulas are programmed into most financial calculators and several spreadsheet functions (such as PV, FV, RATE, NPER, and PMT).[6]

For any of the equations below, the formula may also be rearranged to determine one of the other unknowns. In the case of the standard annuity formula, however, there is no closed-form algebraic solution for the interest rate (although financial calculators and spreadsheet programs can readily determine solutions through rapid trial and error algorithms).

These equations are frequently combined for particular uses. For example, bonds can be readily priced using these equations. A typical coupon bond is composed of two types of payments: a stream of coupon payments similar to an annuity, and a lump-sum return of capital at the end of the bond's maturity - that is, a future payment. The two formulas can be combined to determine the present value of the bond.

An important note is that the interest rate i is the interest rate for the relevant period. For an annuity that makes one payment per year, *i* will be the annual interest rate. For an income or payment stream with a different payment schedule, the interest rate must be converted into the relevant periodic interest rate. For example, a monthly rate for a mortgage with monthly payments requires that the interest rate be divided by 12 (see the example below). See compound interest for details on converting between different periodic interest rates.

The rate of return in the calculations can be either the variable solved for, or a predefined variable that measures a discount rate, interest, inflation, rate of return, cost of equity, cost of debt or any number of other analogous concepts. The choice of the appropriate rate is critical to the exercise, and the use of an incorrect discount rate will make the results meaningless.

For calculations involving annuities, you must decide whether the payments are made at the end of each period (known as an ordinary annuity), or at the beginning of each period (known as an annuity due). If you are using a financial calculator or a spreadsheet, you can usually set it for either calculation. The following formulas are for an ordinary annuity. If you want the answer for the Present Value of an annuity due simply multiply the PV of an ordinary annuity by $(1 + i)$.

2 Formula

The following formula use these common variables:

- PV is the value at time=0 (present value)
- FV is the value at time=n (future value)
- A is the value of the individual payments in each compounding period
- n is the number of periods (not necessarily an integer)
- i is the discount rate, or the interest rate at which the amount will be compounded each period
- g is the growing rate of payments over each time period

2.1 Future value of a present sum

The future value (FV) formula is similar and uses the same variables.

$$FV\ =\ PV\cdot(1+i)^n$$

2.2 Present value of a future sum

The present value formula is the core formula for the time value of money; each of the other formulae is derived from this formula. For example, the annuity formula is the sum of a series of present value calculations.

The present value (PV) formula has four variables, each of which can be solved for:

$$PV\ =\ \frac{FV}{(1+i)^n}$$

The cumulative present value of future cash flows can be calculated by summing the contributions of FVt, the value of cash flow at time t

$$PV\ =\ \sum_{t=1}^{n}\frac{FV_t}{(1+i)^t}$$

Note that this series can be summed for a given value of n, or when n is ∞.[7] This is a very general formula, which leads to several important special cases given below.

2.3 Present value of an annuity for n payment periods

In this case the cash flow values remain the same throughout the n periods. The present value of an annuity (PVA) formula has four variables, each of which can be solved for:

$$PV(A)\ =\ \frac{A}{i}\cdot\left[1-\frac{1}{(1+i)^n}\right]$$

To get the PV of an annuity due, multiply the above equation by $(1 + i)$.

2.4 Present value of a growing annuity

In this case each cash flow grows by a factor of (1+g). Similar to the formula for an annuity, the present value of a growing annuity (PVGA) uses the same variables with the addition of g as the rate of growth of the annuity (A is the annuity payment in the first period). This is a calculation that is rarely provided for on financial calculators.

Where i ≠ g :

$$PV\ =\ \frac{A}{(i-g)}\left[1-\left(\frac{1+g}{1+i}\right)^n\right]$$

Where i = g :

$$PV\ =\ \frac{A\times n}{1+i}$$

To get the PV of a growing annuity due, multiply the above equation by $(1 + i)$.

2.5 Present value of a perpetuity

A perpetuity is payments of a set amount of money that occur on a routine basis and continues forever. When $n \to \infty$, the *PV* of a perpetuity (a perpetual annuity) formula becomes simple division.

$$PV(P) = \frac{A}{i}$$

Present Value of Int Factor Annuity

$$A = P(1 + r/n)^{nt}$$

Example:

Investment P = \$1000

Interest i = 6.90% Compounded Qtrly (4 Times in Year)

Tenure Years n = 5

$= 1000 \times (1 + .069/4)^{(5\ yrs\ \times\ 4\ qtrs\ in\ a\ year)} = 1000 \times (1 + 0.069/4)^{20} \approx 1407.84$

2.6 Present value of a growing perpetuity

When the perpetual annuity payment grows at a fixed rate (g) the value is theoretically determined according to the following formula. In practice, there are few securities with precise characteristics, and the application of this valuation approach is subject to various qualifications and modifications. Most importantly, it is rare to find a growing perpetual annuity with fixed rates of growth and true perpetual cash flow generation. Despite these qualifications, the general approach may be used in valuations of real estate, equities, and other assets.

This is the well known Gordon Growth model used for stock valuation.

2.7 Future value of an annuity

The future value of an annuity (FVA) formula has four variables, each of which can be solved for:

$$FV(A) = A \cdot \frac{(1+i)^n - 1}{i}$$

To get the FV of an annuity due, multiply the above equation by (1 + i).

2.8 Future value of a growing annuity

The future value of a growing annuity (FVA) formula has five variables, each of which can be solved for:

Where i ≠ g :

$$FV(A) = A \cdot \frac{(1+i)^n - (1+g)^n}{i - g}$$

Where i = g :

$$FV(A) = A \cdot n(1+i)^{n-1}$$

2.9 Formula table

The following table summarizes the different formulas commonly used in calculating the time value of money.[8]

Notes:

- *A* is a fixed payment amount, every period
- *G* is a steadily increasing payment amount, that starts at *G* and increases by *G* for each subsequent period.
- *D* is an exponentially or geometrically increasing payment amount, that starts at *D* and increases by a factor of (1+*g*) each subsequent period.

3 Derivations

3.1 Annuity derivation

The formula for the present value of a regular stream of future payments (an annuity) is derived from a sum of the formula for future value of a single future payment, as below, where *C* is the payment amount and *n* the period.

A single payment C at future time *m* has the following future value at future time *n*:

$$FV = C(1+i)^{n-m}$$

Summing over all payments from time 1 to time n, then reversing t

$$FVA = \sum_{m=1}^{n} C(1+i)^{n-m} = \sum_{k=0}^{n-1} C(1+i)^k$$

Note that this is a geometric series, with the initial value being $a = C$, the multiplicative factor being $1 + i$, with n terms. Applying the formula for geometric series, we get

$$FVA = \frac{C(1-(1+i)^n)}{1-(1+i)} = \frac{C(1-(1+i)^n)}{-i}$$

The present value of the annuity (PVA) is obtained by simply dividing by $(1+i)^n$:

$$PVA = \frac{FVA}{(1+i)^n} = \frac{C}{i}\left(1 - \frac{1}{(1+i)^n}\right)$$

Another simple and intuitive way to derive the future value of an annuity is to consider an endowment, whose interest is paid as the annuity, and whose principal remains constant. The principal of this hypothetical endowment can be computed as that whose interest equals the annuity payment amount:

Principal $\times\, i = C$

Principal $= C/i + goal$

Note that no money enters or leaves the combined system of endowment principal + accumulated annuity payments, and thus the future value of this system can be computed simply via the future value formula:

$$FV = PV(1+i)^n$$

Initially, before any payments, the present value of the system is just the endowment principal ($PV = C/i$). At the end, the future value is the endowment principal (which is the same) plus the future value of the total annuity payments ($FV = C/i + FVA$). Plugging this back into the equation:

$$\frac{C}{i} + FVA = \frac{C}{i}(1+i)^n$$

$$FVA = \frac{C}{i}\left[(1+i)^n - 1\right]$$

3.2 Perpetuity derivation

Without showing the formal derivation here, the perpetuity formula is derived from the annuity formula. Specifically, the term:

$$\left(1 - \frac{1}{(1+i)^n}\right)$$

can be seen to approach the value of 1 as *n* grows larger. At infinity, it is equal to 1, leaving $\frac{C}{i}$ as the only term remaining.

4 Examples

4.1 Example 1: Present value

One hundred euros to be paid 1 year from now, where the expected rate of return is 5% per year, is worth in today's money:

$$P = F \times (P/F) = F \times \frac{1}{(1+i)^n} = \frac{100}{1.05} = 95.24$$

So the present value of €100 one year from now at 5% is €95.24.

4.2 Example 2: Present value of an annuity — solving for the payment amount

Consider a 10-year mortgage where the principal amount *P* is $200,000 and the annual interest rate is 6%.

The number of monthly payments is

$$n = 10 \text{ years} \times 12 \text{ months per year} = 120 \text{ months}$$

and the monthly interest rate is

$$i = \frac{6\% \text{ per year}}{12 \text{ months per year}} = 0.5\% \text{ per month}$$

The annuity formula for (*A*/*P*) calculates the monthly payment:

$$A = P\times(A/P) = P\times\frac{i(1+i)^n}{(1+i)^n - 1} = \$200,000\times\frac{0.005(1.005)^{120}}{(1.005)^{120} - 1}$$

$$\approx \$200,000 \times 0.01110205 \approx \$2,220.41 \text{ per month}$$

This is considering an interest rate compounding monthly. If the interest were only to compound yearly at 6%, the monthly payment would be significantly different.

4.2.1 An approximate solution

For those who only want a rough idea of the mortgage payment there is a much less intimidating approximate formula here. For the numbers given above we simply compute an approximate annual repayment of 200,000*(1/n + (2/3)*i) where n=10 yrs, i=0.06. So 200,000*(1/10 + (2/3)*0.06) = 200,000*(0.1+0.04) = 200,000*0.14 = $28,000 per year, roughly, via mental arithmetic alone. Note, as this is an approximation we may ignore the subtleties of monthly compounding. Now $28,000 per year is about 28,000/12 = $2,333 per month which approximates the true answer to within about 5% but has required only mental arithmetic.

4.3 Example 3: Solving for the period needed to double money

Consider a deposit of £100 placed at 10% (annual). How many years are needed for the value of the deposit to double to £200?

Using the algebraic identity that if:

$$x = b^y$$

then

$$y = \frac{\log(x)}{\log(b)}$$

The present value formula can be rearranged such that:

$$y = \frac{\log(\frac{FV}{PV})}{\log(1+i)} = \frac{\log(\frac{200}{100})}{\log(1.10)} = 7.27(years)$$

This same method can be used to determine the length of time needed to increase a deposit to any particular sum, as long as the interest rate is known. For the period of time needed to double an investment, the Rule of 72 is a useful short-cut that gives a reasonable approximation of the period needed.

4.4 Example 4: What return is needed to double money?

Similarly, the present value formula can be rearranged to determine what rate of return is needed to accumulate a given amount from an investment. For example, £100 is invested today and £200 return is expected in five years; what rate of return (interest rate) does this represent?

The present value formula restated in terms of the interest rate is:

$$i = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1 = \left(\frac{200}{100}\right)^{\frac{1}{5}} - 1 = 2^{0.20} - 1 = 0.15 = 15\%$$

see also Rule of 72

4.5 Example 5: Calculate the value of a regular savings deposit in the future.

To calculate the future value of a stream of savings deposit in the future requires two steps, or, alternatively, combining the two steps into one large formula. First, calculate the present value of a stream of deposits of $1,000 every year for 20 years earning 7% interest:

$$PVA = A \cdot \frac{1 - \frac{1}{(1+i)^n}}{i} = 1000 \cdot \frac{1 - \frac{1}{(1+.07)^{20}}}{.07} = 1000 \cdot \frac{1 - 0.258}{.07} = 1$$

This does not sound like very much, but remember - this is *future money* discounted back to its value *today*; it is understandably lower. To calculate the future value (at the end of the twenty-year period):

$$FV = PV(1+i)^n = \$10,594 \times (1+.07)^{20} \approx \$10,594 \times 3.87 = \$40,9$$

These steps can be combined into a single formula:

$$FV = A \cdot \frac{1 - \frac{1}{(1+i)^n}}{i} \cdot (1+i)^n = A \cdot \frac{(1+i)^n - 1}{i}$$

4.6 Example 6: Price/earnings (P/E) ratio

It is often mentioned that perpetuities, or securities with an indefinitely long maturity, are rare or unrealistic, and particularly those with a growing payment. In fact, many types of assets have characteristics that are similar to perpetuities. Examples might include income-oriented real estate, preferred shares, and even most forms of publicly traded stocks. Frequently, the terminology may be slightly different, but are based on the fundamentals of time value of money calculations. The application of this methodology is subject to various qualifications or modifications, such as the Gordon growth model.

For example, stocks are commonly noted as trading at a certain P/E ratio. The P/E ratio is easily recognized as a variation on the perpetuity or growing perpetuity formulae - save that the P/E ratio is usually cited as the *inverse* of the "rate" in the perpetuity formula.

If we substitute for the time being: the *price* of the stock for the present value; the earnings per share of the stock for the cash annuity; and, the discount rate of the stock for the interest rate, we can see that:

$$\frac{P}{E} = \frac{1}{i} = \frac{PV}{A}$$

And in fact, the P/E ratio is analogous to the inverse of the interest rate (or discount rate).

$$\frac{1}{P/E} = i$$

Of course, stocks may have increasing earnings. The formulation above does not allow for growth in earnings, but to incorporate growth, the formula can be restated as follows:

$$\frac{P}{E} = \frac{1}{(i-g)}$$

If we wish to determine the implied rate of growth (if we are given the discount rate), we may solve for *g*:

$$g = i - \frac{E}{P}$$

5 Continuous compounding

Rates are sometimes converted into the continuous compound interest rate equivalent because the continuous equivalent is more convenient (for example, more easily differentiated). Each of the formulæ above may be restated in their continuous equivalents. For example, the present value at time 0 of a future payment at time **t** can be restated in the following way, where **e** is the base of the natural logarithm and **r** is the continuously compounded rate:

$$\text{PV} = \text{FV} \cdot e^{-rt}$$

This can be generalized to discount rates that vary over time: instead of a constant discount rate *r*, one uses a function of time *r*(*t*). In that case the discount factor, and thus the present value, of a cash flow at time T is given by the integral of the continuously compounded rate *r*(*t*):

$$\text{PV} = \text{FV} \cdot \exp\left(-\int_0^T r(t)\,dt\right)$$

Indeed, a key reason for using continuous compounding is to simplify the analysis of varying discount rates and to allow one to use the tools of calculus. Further, for interest accrued and capitalized overnight (hence compounded daily), continuous compounding is a close approximation for the actual daily compounding. More sophisticated analysis includes the use of differential equations, as detailed below.

5.1 Examples

Using continuous compounding yields the following formulas for various instruments:

Annuity

$$PV = \frac{A(1-e^{-rt})}{e^r-1}$$

Perpetuity

$$PV = \frac{A}{e^r-1}$$

Growing annuity

$$PV = \frac{Ae^{-g}(1-e^{-(r-g)t})}{e^{(r-g)}-1}$$

Growing perpetuity

$$PV = \frac{Ae^{-g}}{e^{(r-g)}-1}$$

Annuity with continuous payments

$$PV = \frac{1-e^{(-rt)}}{r}$$

These formulas assume that payment A is made in the first payment period and annuity ends at time t.[9]

6 Differential equations

Ordinary and partial differential equations (ODEs and PDEs) – equations involving derivatives and one (respectively, multiple) variables are ubiquitous in more advanced treatments of financial mathematics. While time value of money can be understood without using the framework of differential equations, the added sophistication sheds additional light on time value, and provides a simple introduction before considering more complicated and less familiar situations. This exposition follows (Carr & Flesaker 2006, pp. 6–7).

The fundamental change that the differential equation perspective brings is that, rather than computing a *number* (the present value *now*), one computes a *function* (the present value now or at any point in *future*). This function may then be analyzed – how does its value change over time – or compared with other functions.

Formally, the statement that "value decreases over time" is given by defining the linear differential operator $\mathcal{L}$ as:

$$\mathcal{L} := -\partial_t + r(t).$$

This states that values decreases (−) over time (∂*t*) at the discount rate (*r*(*t*)). Applied to a function it yields:

$$\mathcal{L}f = -\partial_t f(t) + r(t)f(t).$$

For an instrument whose payment stream is described by $f(t)$, the value $V(t)$ satisfies the inhomogeneous first-order ODE $\mathcal{L}V = f$ ("inhomogeneous" is because one has f rather than 0, and "first-order" is because one has first derivatives but no higher derivatives) – this encodes the fact that when any cash flow occurs, the value of the instrument changes by the value of the cash flow (if you receive a £10 coupon, the remaining value decreases by exactly £10).

The standard technique tool in the analysis of ODEs is the use of Green's functions, from which other solutions can be built. In terms of time value of money, the Green's function (for the time value ODE) is the value of a bond paying £1 at a single point in time *u* – the value of any other stream of cash flows can then be obtained by taking combinations of this basic cash flow. In mathematical terms, this instantaneous cash flow is modeled as a Dirac delta function $\delta_u(t) := \delta(t-u)$.

The Green's function for the value at time *t* of a £1 cash flow at time *u* is

$$b(t;u) := H(u-t) \cdot \exp\left(-\int_t^u r(v)\,dv\right)$$

where H is the Heaviside step function – the notation " $;u$ " is to emphasize that *u* is a *parameter* (fixed in any instance – the time when the cash flow will occur), while *t* is a *variable* (time). In other words, future cash flows are exponentially discounted (exp) by the sum (integral, $\int$) of the future discount rates ($\int_t^u$ for future, *r*(*v*) for discount rates), while past cash flows are worth 0 ($H(u-t)=1$ if $t<u, 0$ if $t>u$), because they have already occurred. Note that the value *at* the moment of a cash flow is not well-defined – there is a discontinuity at that point, and one can use a convention (assume cash flows have already occurred, or not already occurred), or simply not define the value at that point.

In case the discount rate is constant, $r(v) \equiv r$, this simplifies to

$$b(t;u) = H(u-t) \cdot e^{-(u-t)r} = \begin{cases} e^{-(u-t)r} & t<u \\ 0 & t>u, \end{cases}$$

where $(u-t)$ is "time remaining until cash flow".

Thus for a stream of cash flows *f*(*u*) ending by time *T* (which can be set to $T=+\infty$ for no time horizon) the value at time *t*, $V(t;T)$ is given by combining the values of these individual cash flows:

$$V(t;T) = \int_t^T f(u) b(t;u)\,du.$$

This formalizes time value of money to future values of cash flows with varying discount rates, and is the basis of many formulas in financial mathematics, such as the Black–Scholes formula with varying interest rates.

7 See also

- Actuarial science
- Annuity (finance theory)
- Discounted cash flow
- Discounting
- Earnings growth
- Exponential growth
- Finance
- Hyperbolic discounting
- Internal rate of return
- Net present value
- Option time value
- Perpetuity
- Real versus nominal value (economics)
- Time preference

8 Notes

[1] http://www.investopedia.com/articles/03/082703.asp

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