ABUTMENTS

- The Structure upon which the ends of a Bridge rests is referred to as an **Abutment**

- The most common type of Abutment Structure is a Retaining Wall, Although other types of Abutments are also possible and are used

- A retaining wall is used to hold back an earth embankment or water and to maintain a sudden change in elevation.

- Abutment serves following functions
  
  - Distributes the loads from Bridge Ends to the ground
  - Withstands any loads that are directly imposed on it
  - Provides vehicular and pedestrian access to the bridge

- In case of Retaining wall type Abutment bearing capacity and sliding resistance of the foundation materials and overturning stability must be checked
TYPES OF ABUTMENTS

- Sixteenth edition of the AASHTO (1996) standard specification classifies abutments into four types:
  - Stub abutments,
  - partial-depth abutments,
  - full-depth abutments; and
  - Integral abutments.

**Stub Abutment**

**Partial-Depth Abutment**

Partial Depth abutments are located approximately at mid-depth of the front slope of the approach embankment. The higher backwall and wingwalls may retain fill material, or the embankment slope may continue behind the backwall. In the latter case, a structural approach slab or end span design must bridge the space over the fill slope and curtain walls are provided to close off the open area.
**Full-Depth Abutment**

Full-depth abutments are located at the approximate front toe of the approach embankment, restricting the opening under the structure.

**Integral Abutment**
Peck, Hanson Thornburn Classification

A gravity abutment with wing walls is an abutment that consists of a bridge seat, wing walls, back wall, and footing.

A U-abutment is an abutment whose wing walls are perpendicular to the bridge seat.
Spill-through abutment consists of a beam that supports the bridge seat, two or more columns supporting the beam, and a footing supporting the columns. The columns are embedded up to the bottom of the beam in the fill, which extends on its natural slope in front of the abutment.

Pile-bent abutments. A pile-bent abutment with stub wings is another type of spill-through abutment, where a row of driven piles supports the beam.
Other Types of Abutments

(a) Spill Through
(b) Vertical Wall
(c) Vertical Wall with Flared Wingwalls
SELECTION OF ABUTMENTS:
The procedure of selecting the most appropriate type of abutments can be based on the following consideration:

1. Construction and maintenance cost
2. Cut or fill earthwork situation
3. Traffic maintenance during construction
4. Construction period
5. Safety of construction workers
6. Availability and cost of backfill material
7. Superstructure depth
8. Size of abutment
9. Horizontal and vertical alignment changes
10. Area of excavation
11. Aesthetics and similarity to adjacent structures
12. Previous experience with the type of abutment
13. Ease of access for inspection and maintenance.
LIMIT STATES

When abutments fail to satisfy their intended design function, they are considered to reach “limit states.” Limit states can be categorized into two types:

1) ULTIMATE LIMIT STATES.

An abutment reaches an ultimate limit state when:

i.) The strength of at least one of its components is fully mobilized or

ii.) The structure becomes unstable.

In the ultimate limit state an abutment may experience serious distress and structural damage, both local and global. In addition, various failure modes in the soil that supports the abutment can also be identified. These are also called ultimate limit states, they include bearing capacity failure, sliding, overturning, and overall instability.

2) SERVICEABILITY LIMIT STATES.

An abutment experiences a serviceability limit state when it fails to perform its intended design function fully, due to excessive deformation or deterioration. Serviceability limit states include excessive total or differential settlement, lateral movement, fatigue, vibration, and cracking.
LOAD AND PERFORMANCE FACTORS

The AASHTO (1990) bridge specifications require the use of the load and resistance factor design (LRFD) method in the substructure design. A mathematical statement of LRFD can be expressed as

\[ \phi R_n \geq \text{effect of } \Sigma \gamma_i Q_i \]

where

- \( \phi \) = performance or resistance factor
- \( R_n \) = nominal resistance
- \( \gamma_i \) = load factor for load component \( i \)
- \( Q_i \) = load component \( i \)

\[ \phi R_n \geq \text{effect of } \Sigma \gamma_i Q_i \]

- \( \phi \) = performance or resistance factor
- \( R_n \) = nominal resistance
- \( \gamma_i \) = load factor for load component \( i \)
- \( Q_i \) = load component \( i \)

**i) Load Factors:**

Load factors are applied to loads to account for uncertainties in selecting loads and load effects. The load factors used in the first edition of the AASHTO (1994) LRFD bridge specifications are shown in Tables 3.1 and 3.2. of the Text.

**ii) Performance Factors:**

Performance or resistance factors are used to account for uncertainties in structural properties, soil properties, variability in workmanship, and inaccuracies in the design equations used to estimate the capacity. These factors are
used for design are the ultimate limit state suggested values of performance factors for shallow foundations are listed in table 10.2

<table>
<thead>
<tr>
<th>Type of Limit State</th>
<th>Performance Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bearing capacity</td>
<td></td>
</tr>
<tr>
<td>a. Sand</td>
<td></td>
</tr>
<tr>
<td>Semiempirical procedure (SPT)</td>
<td>0.45</td>
</tr>
<tr>
<td>Semiempirical procedure (CPT)</td>
<td>0.55</td>
</tr>
<tr>
<td>Rational method</td>
<td></td>
</tr>
<tr>
<td>Using $\phi_r$ estimated from SPT</td>
<td>0.35</td>
</tr>
<tr>
<td>Using $\phi_r$ estimated from CPT</td>
<td>0.45</td>
</tr>
<tr>
<td>b. Clay</td>
<td></td>
</tr>
<tr>
<td>Semiempirical procedure (CPT)</td>
<td>0.50</td>
</tr>
<tr>
<td>Rational method</td>
<td></td>
</tr>
<tr>
<td>Using shear strength in lab tests</td>
<td>0.60</td>
</tr>
<tr>
<td>Using shear strength from field vane tests</td>
<td>0.60</td>
</tr>
<tr>
<td>Using shear strength estimated from CPT data</td>
<td>0.50</td>
</tr>
<tr>
<td>c. Rock</td>
<td></td>
</tr>
<tr>
<td>Semiempirical procedure</td>
<td>0.60</td>
</tr>
<tr>
<td>2. Sliding</td>
<td></td>
</tr>
<tr>
<td>a. Precast concrete placed on sand</td>
<td></td>
</tr>
<tr>
<td>Using $\phi_r$ estimated from SPT</td>
<td>0.90</td>
</tr>
<tr>
<td>Using $\phi_r$ estimated from CPT</td>
<td>0.90</td>
</tr>
<tr>
<td>b. Concrete cast in place on sand</td>
<td></td>
</tr>
<tr>
<td>Using $\phi_r$ estimated from SPT</td>
<td>0.80</td>
</tr>
<tr>
<td>Using $\phi_r$ estimated from CPT</td>
<td>0.80</td>
</tr>
<tr>
<td>c. Clay (where shear strength is less than 0.5 times normal pressure)</td>
<td>0.85</td>
</tr>
<tr>
<td>Using shear strength in lab</td>
<td></td>
</tr>
<tr>
<td>Using shear strength from field vane test</td>
<td>0.85</td>
</tr>
<tr>
<td>Using shear strength estimated from CPT data</td>
<td>0.80</td>
</tr>
<tr>
<td>d. Clay (where the strength is greater than 0.5 times normal pressure)</td>
<td>0.85</td>
</tr>
</tbody>
</table>
FORCES ON ABUTMENTS

Earth pressures exerted on an abutment can be classified according to the direction and the magnitude of the abutment movement.

1) At-rest Earth Pressure

When the wall is fixed rigidly and does not move, the pressure exerted by the soil on the wall is called at-rest earth pressure.

2) Active Earth Pressure:

When a wall moves away from the backfill, the earth pressure decreases (active pressure).

3) Passive Earth Pressure

When it moves toward the backfill, the earth pressure increases (passive pressure).

Table 10.3, obtained through experimental data and finite element analyses (Clough and Duncan, 1991), gives approximate magnitudes of wall movements required to reach minimum active and maximum passive earth pressure conditions. Observation
<table>
<thead>
<tr>
<th>Type of Backfill</th>
<th>Active</th>
<th>Passive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense sand</td>
<td>0.001</td>
<td>0.01</td>
</tr>
<tr>
<td>Medium dense sand</td>
<td>0.002</td>
<td>0.02</td>
</tr>
<tr>
<td>Loose sand</td>
<td>0.004</td>
<td>0.04</td>
</tr>
<tr>
<td>Compacted silt</td>
<td>0.002</td>
<td>0.02</td>
</tr>
<tr>
<td>Compacted lean clay</td>
<td>0.01&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.05&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>Compacted fat clay</td>
<td>0.01&lt;sup&gt;c&lt;/sup&gt;</td>
<td>0.05&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sup>a</sup> After Clough and Duncan, 1991.
<sup>b</sup> $\Delta$ = movement of top of wall to reach minimum active or maximum passive pressure, by tilting or lateral translation.
<sup>c</sup> $H$ = height of wall.
<sup>c</sup> Under stress conditions close to the minimum active or maximum passive earth pressures, cohesive soils creep continually. The movements shown would produce active or passive pressures only temporarily. With time the movements would continue if pressures remain constant. If movement remains constant, active pressures will increase with time, approaching the at-rest pressure, and passive pressures will decrease with time, approaching values on the order of 40% of the maximum short-term passive pressure.
1. The required movements for the extreme conditions are approximately proportional to the wall height.

2. The movement required to reach the maximum passive pressure is about 10 times as great as that required to reach the minimum active pressure for walls of the same height.

3. The movement required to reach the extreme conditions for dense and incompressible soils is smaller than those for loose and compressible soil.

For any cohesionless backfill, conservative and simple guidelines for the maximum movements required to reach the extreme cases are provided by Clough and Duncan (1991).

*For minimum active pressure, the movements no more than about 1 mm in 240 mm (Δ/H = 0.004) and for maximum passive pressure about 1 mm in 24 mm (Δ/H = 0.004).*

As shown in [figure 10.10](#):

- The value for the earth pressure coefficient varies with wall displacement and eventually remains constant after sufficiently large displacements.
- The change of pressures also varies with the type of soil, that is, the pressures in the dense sand change more quickly with wall movement.
Dense Sand, $\phi_r = 45^\circ$, $K_p = 5.8$

Medium Dense Sand, $\phi_r = 37^\circ$, $K_p = 4.0$

Loose Sand, $\phi_r = 30^\circ$, $K_p = 3.0$

**Passive Movement**

\[ K_0 = 1 - \sin \phi_r = 0.50 \]
\[ K_0 = 1 - \sin \phi_r = 0.40 \]
\[ K_0 = 1 - \sin \phi_r = 0.29 \]

**Active Movement**

\[ K_0 = 0.5 \]
\[ 0.25 \]
\[ 0.12 \]

**WALL MOVEMENT / WALL HEIGHT - $\Delta$/H**

**Fig. 10.10** Relationship between wall movement and earth pressure. [After Clough and Duncan, 1991.]
Note: Average $K$ after compaction varies with compaction procedure and height of wall.

Medium Dense Sand, $\phi_r = 37^\circ$, $K_p = 4.0$

* Compaction develops higher pressures only on stiff walls on non-yielding foundations. Compaction of backfill behind flexible walls or walls on yielding foundations causes them to move and the earth pressures are lower than for stiff walls.

$K_o = 1.0$ average after compaction

$K_o = 1 - \sin \phi_r = 0.40$

Ignoring effects of compaction

$\phi_r = 0.37^\circ$, $K_o = 0.25$

$\Delta$

$H$

Passive Movement

$\Delta$

$H$

Active Movement

Fig. 10.11 Relationship between wall movement and earth pressure for a wall with compacted backfill. [After Clough and Duncan, 1]
METHODS FOR ESTIMATING $K_A$ AND $K_P$

Coulomb in 1776 and Rankine in 1856 developed simple methods for calculating the active and passive earth pressures exerted on retaining structures. Caquot and Kerisel (1948) developed the more generally applicable log spiral theory, where the movements of walls are sufficiently large so that the shear strength of the backfill soil is fully mobilized, and where the strength properties of the backfill can be estimated with sufficient accuracy, these methods of calculation are useful for practical purposes.

Coulomb’s trial wedge method can be used for irregular backfill configurations and Rankine’s theory and the log spiral analysis can be used for more regular configurations. Each of these methods will be discussed below.

COULOMB THEORY:
The coulomb theory, the first rational solution to the earth pressure problem, is based on the concept that the lateral force exerted on a wall by the backfill can be evaluated by analysis of the equilibrium of a wedge-shaped mass of soil bounded by the back of the wall, the backfill surface, and a surface of sliding through the soil. The assumptions in this analysis are
1. The surface of sliding through the soil is a straight line.
2. The full strength of the soil is mobilized to resist sliding (shear failure) through the soil.

i) **Active Pressure**: A graphical illustration for the mechanism for active failure according to the coulomb theory is shown in Figure 10.12a. The active earth pressure force can be expressed as:
$P_a = \text{active earth pressure force (force/length)}$

$= \frac{1}{2} \gamma H^2 K_a$

$K_a = \text{coefficient of active earth pressure}$

$\gamma = \text{unit weight of backfill soil (force/length}^3)$

$H = \text{wall height (length)}$

$\phi_f = \text{the internal friction angle of soil (degrees)}$

$\beta = \text{the slope of stem face (degrees)}$

$\delta = \text{the friction angle between wall and soil (degrees)}$

$i = \text{the slope of backfill surface (degrees)}$

$$P_a = \frac{1}{2} \gamma H^2 \frac{\cos^2(\phi_f - \beta)}{\cos^2 \beta \cos(\beta + \delta) [1 + \frac{\sin(\phi_f + \delta) \sin(\phi_f - i)}{\cos(\beta + \delta) \cos(\beta - i)}]^2}$$ (10.2)
**Passive Pressure:**

The coulomb theory can be used to evaluate passive resistance, using the same basic assumptions. **Figure 10.12b** shows the failure mechanism for the passive case. The passive earth pressure force, $P_p$, can be expressed as follows:

\[
P_p = \frac{1}{2} \gamma H^2 \frac{\cos^2(\phi_f + \beta)}{\cos^2 \beta \cos(\beta - \delta) \left[ 1 - \frac{\sin(\phi_f + \delta) \sin(\phi_f + \delta)}{\sqrt{\cos(\beta - \delta) \cos(\beta - \delta)}} \right]^2} \tag{10.3}
\]

**Fig. 10.12** Coulomb theory for active and passive earth pressures.
The basic assumption in the coulomb theory is that the surface of sliding is a plane. This assumption does not affect appreciably the accuracy for the active case. However, for the passive case, values of $p_p$ calculated by the coulomb theory can be much larger than can actually be mobilized, especially when the value of $\delta$ exceeds about one half of $\varphi_f$.

Wall Friction: friction between the wall and backfill has an important effect on the magnitude of earth pressures and an even more important effect on the direction of the earth pressure force.

Table 10.4 presents values of the maximum possible wall friction angle for various wall materials and soil types.
RANKINE THEORY: The Rankine theory is applicable to conditions where the wall friction angle (\(\varphi\)) is equal to the slope of the backfill surface (I). As in the case of the coulomb theory, it is assumed that the strength of the soil is fully mobilized.

Table 10.4

<table>
<thead>
<tr>
<th>Interface Materials</th>
<th>Friction Angle, (\delta) (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass concrete on the following foundation materials</td>
<td>35</td>
</tr>
<tr>
<td>Clean gravel, gravel-sand mixtures, coarse sand</td>
<td>29–31</td>
</tr>
<tr>
<td>Clean fine to medium sand, silty medium to coarse sand, silty or clayey gravel</td>
<td>24–29</td>
</tr>
<tr>
<td>Clean fine sand, silty or clayey fine to medium sand</td>
<td>19–24</td>
</tr>
<tr>
<td>Fine sandy silt, nonplastic silt</td>
<td>17–19</td>
</tr>
<tr>
<td>Very stiff and hard residual or preconsolidated clay</td>
<td>22–26</td>
</tr>
<tr>
<td>Medium stiff and stiff clay and silty clay</td>
<td>17–19</td>
</tr>
<tr>
<td>Masonry on foundation materials has same friction factors</td>
<td></td>
</tr>
<tr>
<td>Steel sheet piles against the following soils</td>
<td>22</td>
</tr>
<tr>
<td>Clean gravel, gravel–sand mixtures, well-graded rock fill with spalls</td>
<td>17</td>
</tr>
<tr>
<td>Clean sand, silty sand–gravel mixture, single-size hard rock fill</td>
<td></td>
</tr>
<tr>
<td>Silty sand, gravel or sand mixed with silt or clay</td>
<td>14</td>
</tr>
<tr>
<td>Fine sandy silt, nonplastic silt</td>
<td>11</td>
</tr>
<tr>
<td>Formed or precast concrete or concrete sheet piling against the following soils</td>
<td>22–26</td>
</tr>
<tr>
<td>Clean gravel, gravel–sand mixture, well-graded rock fill with spalls</td>
<td>17–22</td>
</tr>
<tr>
<td>Clean sand, silty sand–gravel mixture, single-size hard rock fill</td>
<td></td>
</tr>
<tr>
<td>Silty sand, gravel or sand mixed with silt or clay</td>
<td>17</td>
</tr>
<tr>
<td>Fine sandy silt, nonplastic silt</td>
<td>14</td>
</tr>
<tr>
<td>Various structural materials</td>
<td></td>
</tr>
<tr>
<td>Masonry on masonry, igneous, and metamorphic rocks:</td>
<td></td>
</tr>
<tr>
<td>Dressed soft rock on dressed soft rock</td>
<td>35</td>
</tr>
<tr>
<td>Dressed hard rock on dressed soft rock</td>
<td>33</td>
</tr>
<tr>
<td>Dressed hard rock on dressed hard rock</td>
<td>29</td>
</tr>
<tr>
<td>Masonry on wood in direction of cross grain</td>
<td>26</td>
</tr>
<tr>
<td>Steel on steel at sheet pile interlocks</td>
<td>17</td>
</tr>
</tbody>
</table>

*From U.S. Dept. of Navy, 1982b.*
i) Active Pressure:

The active earth pressure considered in the Rankine theory is illustrated in Figure 10.13a for a level backfill condition. The coefficient of active earth pressure, \( k_a \), can be expressed as:

\[
K_a = \cos i \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi_f}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi_f}}
\]  

(10.4)

When the ground surface is horizontal, that is, when \( I = 0 \), \( k_a \) can be expressed as

\[
P_a = \text{the active pressure (force/length}^2)\),
\]

\[
K_a = \text{the active pressure coefficient}
\]

\[
\gamma = \text{the unit weight of soil (force/length}^3\)
\]

\[
c = \text{the cohesion (force/length}^2\)
\]

\[
z = \text{the depth below the ground surface (length)}
\]

The variation of active pressure with depth is linear, as shown in figure 10.13b. If the backfill is cohesive, the soil is theoretically in a tension zone down to a depth of \( 2c/\gamma (k_a)^2 \). However, a tension crack is likely to develop in that zone and may be filled with water, so that hydrostatic pressure will be exerted on the wall, as shown in figure 10.13c.

ii) Passive Pressure: The Rankine theory can also be applied to passive pressure conditions. The passive earth pressure coefficient (\( k_p \)) can be expressed as
\[ K_p = \cos \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi_f}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi_f}} \] (10.7)

When the ground surface is horizontal, \( K_p \) can be expressed as

\[ K_p = \frac{1 + \sin \phi_f}{1 - \sin \phi_f} \] (10.8)

The passive pressure at depth \( z \) can be expressed as

\[ P_p = K_p \gamma z + 2c\sqrt{K_p} \] (10.9)

where

\( P_p \) = the passive pressure (force/length^2)
\( K_p \) = the passive pressure coefficient
Fig. 10.13  Rankine theory for active pressure, frictionless wall [After Clough and Duncan, 1991.]
**LOG SPIRAL ANALYSIS:**

The failure surface in most cases is more closely approximated by a log spiral than a straight line, as shown in figure 10.14. Active and passive pressure coefficients, $K_a$ and $k_p$, obtained from analysis using log spiral surfaces are listed in tables 10.5 and 10.6 (Caquot and Kerisel, 1948). Values of $K_a$ and $k_p$ for walls with level backfill and vertical stem also shown in figure 10.15. These values are also based on the log spiral analyses performed by Caquot and Kerisel.

![Log Spiral Failure Surface](image)
<table>
<thead>
<tr>
<th>$\delta$ (deg)</th>
<th>$\iota$ (deg)</th>
<th>$\beta$ (deg)</th>
<th>$\phi_f$ (deg)</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15</td>
<td>-10</td>
<td>0.37</td>
<td>0.30</td>
<td>0.24</td>
<td>0.19</td>
<td>0.14</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.42</td>
<td>0.35</td>
<td>0.29</td>
<td>0.24</td>
<td>0.19</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.45</td>
<td>0.39</td>
<td>0.34</td>
<td>0.29</td>
<td>0.24</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
<td>0.42</td>
<td>0.34</td>
<td>0.27</td>
<td>0.21</td>
<td>0.16</td>
<td>0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.49</td>
<td>0.41</td>
<td>0.33</td>
<td>0.27</td>
<td>0.22</td>
<td>0.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.55</td>
<td>0.47</td>
<td>0.40</td>
<td>0.34</td>
<td>0.28</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-10</td>
<td>0.55</td>
<td>0.41</td>
<td>0.32</td>
<td>0.23</td>
<td>0.17</td>
<td>0.13</td>
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<td>0.34</td>
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<td>-15</td>
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<td>0.31</td>
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<td>0.21</td>
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<tr>
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<td></td>
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<tr>
<td>0</td>
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<td>0.24</td>
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<tr>
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<td>0.30</td>
<td>0.26</td>
<td>0.22</td>
<td>0.19</td>
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*After Caquot and Kerisel, 1948.*
### TABLE 10.6 Values of $K_p$ for Log Spiral Failure Surface

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<th>$\phi_f$ (deg)</th>
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</tbody>
</table>

*After Caquot and Kerisel, 1948.*
Fig. 10.15  Active and passive pressure coefficients for vertical wall and horizontal backfill—based on log spiral failure surfaces [After Caquot and Kerisel, 1948.]
SELECTION OF EARTH PRESSURE COEFFICIENTS:
Selecting a proper earth pressure coefficient is essential for successful wall design. A number of methods previously discussed can be used to decide the magnitude of the coefficients. A decision on what type of earth pressure coefficient should be used is based on the direction and the magnitude of the wall movement.

The New Zealand Ministry of Works and Development (NAMWD, 1979) has recommended the following static earth pressure coefficients for use in design:

1. Counterfort or gravity walls founded on rock or piles: $K_0$.
2. Cantilever walls less than 1880-mm high founded on rock or piles: $(K_0 + K_a)/2$.
3. Cantilever walls higher than 4880-mm or any wall founded on a spread footing: $K_a$.

LOCATION OF HORIZONTAL RESULTANT:
In conventional designs and analyses, the horizontal resultant is assumed to be located at one-third of total height from the bottom of the wall. However, several experimental tests performed by researchers conclude that the resultant is applied at 0.40H to 0.45H from the bottom of the wall where H is the total height of the wall.

EQUIVALENT FLUID PRESSURE:
Equivalent fluid pressures provide a convenient means of estimating design earth pressures, especially when the backfill material is a clayey soil. The lateral earth pressure at depth $z$ can be expressed as
Some typical equivalent fluid unit weights and corresponding pressure coefficients are presented in Table 10.7. These are appropriate for use in designing walls up to about 6100mm in height. Values are presented for at rest condition and for walls that can tolerate movements of 1mm in 240mm, and for level and sloped backfill.

When the equivalent fluid pressure is used in the estimation of horizontal earth pressure it is necessary to include vertical earth pressure acting on the wall to avoid an assumption that is too conservative. In the level backfill, the amount of the vertical earth pressure acting on the wall can be taken as much as 10% of the soil weight.
Effect of Surcharges:

When vertical loads act on a surface of the backfill near a retaining wall or an abutment, the lateral and vertical earth pressure used for the design of the wall should be increased.

Uniform Surchage Load:

A surcharge load uniformly distributed over a large ground surface area increases both the vertical and lateral pressures. The increase in the vertical pressure, $\Delta P_v$, is the same as the applied surcharge pressure, $q_s$, that is,

$$\Delta P_v = q_s$$

and the amount of increase in the lateral pressure, $\Delta P_h$, is

$$\Delta P_h = kq_s$$

Where

- $k = \text{an earth pressure coefficient (dimensionless)}$
- $k = ka$ for active pressure
- $k = k_0$ for at-rest condition
- $k = k_p$ for passive pressure

Because the applied area is infinitely large, the increases in both vertical and horizontal pressures are constant over the height of the
Therefore, the horizontal resultant force due to a surcharge load is located at mid height of the wall.

*Point Load and Strip Loads:*
The theory of elasticity can be used to estimate the increased earth pressures induced by various types of surcharge loads. Equations for earth pressures due to point load and strip loads are presented in Figure 10.16.
$$\sigma_h = \frac{P}{\pi R^2} \left[ \frac{3x^2 z}{R^3} - \frac{(1-2v) R}{R+z} \right]$$

where $P =$ force

$$r = \sqrt{x^2 + y^2}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$v =$ Poisson's ratio

$$\sigma_h = \frac{2p}{\pi} \left[ \alpha - \sin \alpha \cos (\alpha + 2\delta) \right]$$

where $p =$ pressure

$$\sigma_h = \frac{2p \nu \alpha}{\pi}$$

Fig. 10.16  Earth pressure due to point load and strip loads.
EQUIVALENT HEIGHT OF SOIL FOR LIVE LOAD SURCHARGE:
In the AASHTO (1994) LRFD Bridge Specifications, the live load surcharge, \( L_S \), is specified in terms of an equivalent height of soil, \( h_{eq} \), representing the vehicular loading. The values specified for \( h_{eq} \) with the height of the wall and are given in Table 10.8.

<table>
<thead>
<tr>
<th>Wall Height (mm)</th>
<th>( h_{eq} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \leq 1500 )</td>
<td>1700</td>
</tr>
<tr>
<td>3000</td>
<td>1200</td>
</tr>
<tr>
<td>6000</td>
<td>760</td>
</tr>
<tr>
<td>( \geq 9000 )</td>
<td>610</td>
</tr>
</tbody>
</table>

*In AASHTO Table 3.11.6.2-1. [From AASHTO LRFD Bridge Design Specifications. Copyright © 1994 by the American Association of State and Highway and Transportation Officials, Washington, DC. Used by permission.]
DESIGN REQUIREMENTS FOR ABUTMENTS

Failure Modes for Abutments:

Abutments are subject to various limit states or types of failure, as illustrated in figure 10.17. Failures can occur within soils or the structural members.

i) Sliding failure occurs when the lateral earth pressure exerted on the abutment exceeds the frictional sliding capacity of the foundation.

ii) If the bearing pressure is larger than the capacity of the foundation soil or rock, bearing failure results.

iii) Deep-seated sliding failure may develop in clayey soil.

iv) Structural failure also should be checked.
Fig. 10.17 Failure modes of abutments.
BASIC DESIGN CRITERIA FOR ABUTMENTS:

For design purposes, abutments on spread footings can be classified into three categories (Duncan et al 1990).

1. Abutment with clayey soils in the backfill or foundations.
2. Abutment with granular backfill and foundations of sand or gravel.
3. Abutment with granular backfill and foundations on rock.

For each category, design procedures and stability criteria for the ASD method and the LRFD method are summarized in Figures 10.18-10.20.
Earth Loads

\[ P_v \] and \[ P_h \] based on experience, with allowance for creep

\[ y = 0.4 \, H \]

Stability Criteria

**ASD Method**

1. \( N \) within middle third of base
2. \( R_t q_{u,ax} / FS \geq q_{max} \) (unfactored)
3. Safe against sliding
   \[ F_r / FS \geq P_h \) (unfactored)\]
4. Settlement within tolerable limits
5. Safe against deep-seated foundation failure

**LRFD Method**

1. \( N \) within middle half of base
2. \( \phi R_t q_{u,ax} \geq q_{u,ax} \)
3. Safe against sliding
   \[ \phi_s F_r \geq \sum \gamma_i P_{ni} \]
4. Settlement within tolerable limits
5. Safe against deep-seated foundation failure

(a) Forces on Wall

(b) Forces on Vertical Plane Through Heel of Wall

Earth Loads

\( P_s \) and \( P_h \) calculated using Coulomb active earth pressure theory
\( \delta \) or \( P_s \) estimated using judgement, with allowance for movement
of backfill relative to wall.

\[ y = 0.4 \, H \]

Stability Criteria

**ASD Method**

or

**LRFD Method**

1. \( N \) within middle third of base
2. \( R_c \, q_u / \text{FS} \geq q_{\text{max}} \) (unfactored)
3. Safe against sliding
   \( F_r / \text{FS} \geq P_h \) (unfactored)
4. Settlement within tolerable limits

Fig. 10.19 Earth loads and stability criteria for walls with granular backfills and
foundations on sand or gravel. [After J. M. Duncan, G. W. Clough, and R. M. Ebetling
Conference on Design and Performance of Earth Retaining Structures, ASCE, Cornell
University, Ithaca, NY. Reproduced by permission of ASCE.]
Earth Loads

- $P_h$ based on at-rest pressure
- $P_e$, estimated using judgement
- $y = 0.4 \, H$

Stability Criteria

**ASD Method**

1. $N$ within middle half of base
2. $R_1 \frac{q_{ul}}{FS} \geq q_{\text{max (unfactored)}}$
3. Safe against sliding: $F_r / FS \geq P_{e(\text{unfactored})}$

**LRFD Method**

1. $N$ within middle three quarters of base
2. $\phi \, R_1 \frac{q_{ul}}{FS} \geq q_{u,\text{max}}$
3. Safe against sliding: $\phi \, F_r \geq \Sigma \gamma_i \, P_{ie}$

Fig. 10.20 Earth loads and stability criteria for walls with granular backfills and foundations on rock. [After J. M. Duncan, G. W. Clough, and R. M. Ebeling (1990), "Behavior and Design of Gravity Earth Retaining Structures," *Proceedings of Conference on Design and Performance of Earth Retaining Structures*. ASCE, Cornell University, Ithaca, NY. Reproduced by permission of ASCE.]
PROCEDURE FOR DESIGN OF ABUTMENTS:

A series of steps must be followed to obtain a satisfactory design.

STEP 1: SELECT PRELIMINARY PROPORTIONS OF THE WALL.
STEP 2: DETERMINE LOADS AND EARTH PRESSURES.
STEP 3: CALCULATE MAGNITUDE OF REACTION FORCES ON BASE.
STEP 4: CHECK STABILITY AND SAFETY CRITERIA
   a. Location of normal component of reactions.
   b. Adequacy of bearing pressure.
   c. Safety against sliding.
STEP 5: REVISE PROPORTIONS OF WALL AND REPEAT STEPS 2-4 UNTIL STABILITY CRITERIA IS SATISFIED AND THEN CHECK
   a. Settlement within tolerable limits.
   b. Safety against deep-seated foundation failure.
STEP 6: IF PROPORTIONS BECOME UNREASONABLE, CONSIDER A FOUNDATION SUPPORTED ON DRIVEN PILES OR DRILLED SHAFTS.
STEP 7: COMPARE ECONOMICS OF COMPLETED DESIGN WITH OTHER SYSTEMS.
STEP 1: SELECT PRELIMINARY PROPORTIONS OF THE WALL.

Figure 10.21 shows commonly used dimensions for a gravity-retaining wall and a cantilever wall. These proportions can be used when scour is not a concern to obtain dimensions for a first trial of the abutment.

Fig. 10.21 Preliminary dimensions for gravity walls and cantilever walls. [After Clayton and Milititsky, 1986.]
STEP 2: DETERMINE LOADS AND EARTH PRESSURES.

Design loads for abutments are obtained by using group load combinations described in Tables 3.1 and 3.2. Methods for calculating earth pressures exerted on the wall are discussed in section 10.4.5. The use of equivalent fluid pressures presented in Table 10.7 gives satisfactory earth pressures if conditions are no unusual.

STEP 3: CALCULATE MAGNITUDE OF REACTION FORCES ON BASE.

Figure 10.22 illustrates a typical cantilever wall subjected to various loads causing reaction forces which are normal to the base (N) and tangent to the base (Fr). These reaction forces are determined by simple static for each load combination being investigated.
Fig. 10.22  Forces on a typical retaining wall or abutment.
STEP 4: CHECK STABILITY AND SAFETY CRITERIA

a. Location of normal component of reactions.
b. Adequacy of bearing pressure.
c. Safety against sliding.

1. The location of the resultant on the base is determined by balancing moments about the toe of the wall. The criteria for foundation on soil for the location of the resultant is that “it must lie within the middle half for LRFD (Figs. 10.18 and 10.19). “ This criterion replaces the check on the ratio of stabilizing moment to overturning moment.

For foundations on rock, the acceptable location of the resultant has a greater range than for foundations on soil “Middle three quarters of base”

As shown in figure 10.23, the location of the resultant, X₀, is obtained by

\[ X₀ = \frac{(\text{Summation of moments about point o})}{N} \]

Where N = the vertical resultant force (force/length).

The eccentricity of the resultant, e, with respect to the centerline of the base is

\[ e = \frac{B}{2} – X₀ \]

where B = base width (length)
2. Safety against bearing failure is obtained by applying a performance factor to the ultimate bearing capacity in the LRFD method. The ultimate BC can be calculated from the in-situ tests or semiempirical procedures.

Safety against bearing failure is checked by

\[ \phi R_i q_{ult} \geq q_{umax} \]

\( q_{ult} \) = ultimate BC (force/length)

\( R_i = \) reduction factor due to inclined loads = \((1 - H_n/V_n)^3\)

\( H_n = \) unfactored horizontal force

\( V_n = \) unfactored vertical force

\( \phi = \) performance or resistance factor

\( q_{umax} = \) maximum bearing pressure due to factored loads (force/length\(^2\))

Shape of Bearing Pressure Distribution:

The resultant, \( N \), will pass through the centered of a triangular or trapezoidal stress distribution, or the middle of a uniformly distributed stress block.

Maximum Bearing Pressure:

The following equations are used to compute the max. soil pressures, \( q_{umax} \) per unit length of a rigid footing.

For a triangular shape of bearing pressure:

When the resultant is within the middle third of base

\[ q_{umax} = \frac{N_u}{B} - \frac{6 N(u) e}{B^2} \]

When the resultant is outside of the middle third of base

\[ q_{umax} = \frac{2 N(u)}{3 X_0} \]
For a uniform distribution of the bearing pressure

\[ q_{\text{umax}} = \frac{N(u)}{2X_0} \]

Where

\( N(u) = \) unfactored (factored) vertical resultant (force/length)
\( X_0 = \) location of the resultant measured from toe (length)
\( e = \) eccentricity of \( N(u) \) (length)
Fig. 10.23 Various shapes of stress distributions and maximum bearing pressures.
3. In the LRFD method, sliding stability is checked by

\[ \phi_s F_{ru} \geq \sum \gamma_i P_{hi} \]

where

\( \phi_s \) = performance factor for sliding (values given in tab 10.2)

\( F_{ru} \) = \( N(u) \tan \delta b + c_a B_e \)

\( Nu \) = factored vertical resultant

\( \delta b \) = friction angle between base and soil

\( c_a \) = adhesion (force/length²)

\( B_e \) = effective length of base in compression

\( \gamma_i \) = load factor for force component \( i \)

\( P_{hi} \) = horizontal earth pressure force \( i \) causing sliding (force/length)

The passive earth pressure generated by the soil in front of the wall may be included to resist sliding if it is ensured that the soil in front of the wall will exist permanently. However, sliding failure occurs in many cases before the passive earth pressure is fully mobilized. Therefore, it is safer to ignore the effect of the passive earth pressure.
STEP 5: REVISE PROPORTIONS OF WALL AND REPEAT STEPS 2-4 UNTIL STABILITY CRITERIA IS SATISFIED AND THEN CHECK

a. Settlement within tolerable limits.

b. Safety against deep-seated foundation failure.

When the preliminary wall dimensions are found inadequate the wall dimensions should be adjusted by a trial and error method.

A sensitivity study done by Kim shows that the stability can be improved by varying the location of the wall stem, the base width, and the wall height. Some suggestions for correcting each stability or safety problems are presented as follows:

1. Bearing failure or eccentricity criterion not satisfied
   a. Increase the base width.
   b. Relocate the wall stem by moving towards the heel.
   c. Minimize Ph by replacing a clayey backfill with granular material or by reducing pore water pressure behind the wall stem with a well designed drainage system.
   d. Provide an adequately designed reinforced concrete approach slab supported at one end by the abutment so that no horizontal pressure due to live load surcharge need be considered.

2. Sliding stability criteria not satisfied
   a. Increase the base width
   b. Minimize Ph as described above
   c. Use an inclined base (heel side down) to increase horizontal distance.
   d. Provide an adequately designed approach slab mentioned above.
   e. Use a shear key

3. Settlement and Overall Stability Check.

Once the proportions of the wall have been selected to satisfy the bearing pressure, eccentricity, and sliding criteria then the requirements on settlement and overall slope stability must be checked.
a. Settlement should be checked for walls founded on compressible soils to ensure that the predicted settlement is less than the settlement than the wall or structure it supports can tolerate. The magnitude of settlement can be estimated using the methods described in the Engineering manual for shallow foundations.

b. The overall stability of slopes with regard to the most critical sliding surface should be evaluated if the wall is underlain by weak soil. This check is based on limiting equilibrium methods, which employ the modified Bishop, simplified Janbu or Spencer analysis.

STEP 6: IF PROPORTIONS BECOME UNREASONABLE, CONSIDER A FOUNDATION SUPPORTED ON DRIVEN PILES OR DRILLED SHAFTS.

Driven piles and drilled shafts can be used when the configuration of the wall is unreasonable or uneconomical.

STEP 7: COMPARE ECONOMICS OF COMPLETED DESIGN WITH OTHER SYSTEMS.

When a design is completed, it should be compared with other types of walls that may result in a more economical design.
Example 10.4.7: Abutment design

Using LRFD method, the stability and safety for the abutment below is to be checked. The abutment is found on sandy gravel with an average SPT blow count of 22. The ultimate bearing capacity (10 tons/sft).

**Fig. 10.24** A design example of bridge abutment.
DETERMINATION OF LOADS AND EARTH PRESSURES

Loadings: The loadings from the superstructure are given as
DL= dead load = 109.4 kN / m
LL= live load = 87.5 kN / m
WS= wind load on superstructure = 2.9 kN / m
WL = wind load on superstructure = 0.7 kN / m
BR= 3.6 kN / m
CR +SH+TU = creep, shrinkage, and temperature = 10% of DL = 10.9 kN / m

Pressures generated by the live load and dead load surcharges can be obtained as
ωL = h_eq γ = 1195 mm x 18.9 kN / m³ = 22.6 kN / m²
ωD = (slab thickness) γc = 305 mm x 23.6 kN / m³ = 7.2 kN / m²
HL = K ωL H’ = 0.25 x 22.6 kN / m² x 2743 mm = 15.51 kN / m
HD = K ωD H’ = 0.25 x 7.2 kN / m² x 2743 mm = 4.94 kN / m
VL = ωL * (heel width) = 22.6 kN / m² x 380 mm = 8.59 kN / m
VD = ωD * (heel width) = 7.2 kN / m² x 380 mm = 2.74kN / m

Pressures due to equivalent fluid pressure can be calculated as
Ph = (½) (EFPh) H² = (½) (5.50)(2.745)² = 20.72 kN / m
Pv= (½) (EFPv H² = (½) (1.89)(2.745)² = 7.12 kN /" m
**Load Combinations**  From Table 3.1 [Table A3.4.1-1], the relevant load combinations are determined to be Strength I and Strength III. Considering the minimum and maximum load factors for permanent loads as shown in Table 3.2 [Table A3.4.1-2], load combinations can be expanded to four groups: Strength I, Strength Ia, Strength III, and Strength IIIa. The load factors and load combinations are summarized as follows:

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<th>EV</th>
<th>EH</th>
<th>LL</th>
<th>BR</th>
<th>LS</th>
<th>WS</th>
<th>WL</th>
<th>CR + SH + TU</th>
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</table>
Fig. 10.25  Loadings applied to the abutment.
### Unfactored Loads

Unfactored vertical and horizontal loads are summarized as follows:

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<th>Vertical Loads</th>
<th>$V_n$ (kN)</th>
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<th>Moment (kN m)</th>
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<tr>
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### Horizontal Loads

$H_n$ (kN) | Arm (mm) | Moment (kN m)
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### Design Loads

Factored design loads are summarized as follows:

#### Vertical Loads, $V_u$(kN/m)

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<td>DC</td>
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#### Moment due to $V_u$(kN m/m)

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## Horizontal Loads, $H_x$(kN/m)

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## STABILITY AND SAFETY CRITERIA

Three design criteria should be satisfied: Eccentricity, Sliding, and Bearing Capacity. The last column of each table represents the design margin which is expressed as

$$\text{Design margin (\%)} = \frac{\text{provided} - \text{applied}}{\text{provided}} \times 100$$

### Eccentricity

In the LRFD method, the eccentricity design criterion is ensured by keeping the resultant force within the middle half of the base width. In other words, the eccentricity should not exceed the maximum eccentricity, $e_{\text{max}} (= B/4)$ in soil foundation. The results are summarized as follows:

<table>
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<th>V_L</th>
<th>H_L</th>
<th>M_V</th>
<th>M_H</th>
<th>X_v</th>
<th>e</th>
<th>$e_{\text{max}}$</th>
<th>Design Margin (%)</th>
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<td>765.61</td>
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<td>457.50</td>
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$$\text{where \%} = \frac{(e_{\text{max}} - e)}{e_{\text{max}}} \times 100$$
Sliding  The results of sliding design criterion are summarized as follows:

<table>
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<tr>
<th>Strength</th>
<th>$V_L$</th>
<th>$\tan \delta_r$</th>
<th>$F_r$</th>
<th>$\phi_z$</th>
<th>$\phi_z F_r$</th>
<th>$H_L$</th>
<th>Design Margin (%)</th>
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<td>79.05</td>
<td>48.00</td>
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</table>

where $\% = (\phi_z F_r - H_L) / (\phi_z F_r) \times 100$

Bearing Capacity  The results of bearing capacity criterion are summarized as follows:

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<tr>
<th>Strength</th>
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<th>$V_L$</th>
<th>$H_L/V_L$</th>
<th>$R_f$</th>
<th>$q_{ult}$</th>
<th>$R_f q_{ult}$</th>
<th>$\phi R_f q_{ult}$</th>
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<td>0.19</td>
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<td>542.72</td>
<td>244.22</td>
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</table>

where $\% = (\phi R_f q_{ult} - q_{max}) / (\phi R_f q_{ult}) \times 100$

CONCLUSIONS

Strength $I$ governs the design with the bearing capacity criterion. If any design criterion is not satisfied, the abutment dimensions should be adjusted by increasing the base width, moving the stem location or changing wall thickness. Because an abutment is subject to numerous loadings and various limit states, it is not clear which dimensions should be adjusted to find the optimum design. Using a spreadsheet program may be one of the most effective ways to design the abutment.
SEISMIC DESIGN OF ABUTMENTS

- The Method most commonly used for Seismic Analysis of Free Standing Abutments is the one Proposed in 1920’s by Mononobe and Okabe

- The method is an Extension of Coulomb Wedge Theory, and takes into account the horizontal and vertical forces that act on the sliding soil wedge

- The assumptions inherent in the theory are:
  - The abutment is free to yield sufficiently so that the Active and passive conditions are realized
  - The backfill is cohesionless with internal friction angle = \( \phi \)
  - The backfill is unsaturated so that liquefaction problems do not arise
MONONOBE – OKABE THEORY FOR SEISMIC DESIGN OF ABUTMENTS
MONONOBE – OKABE THEORY FOR SEISMIC DESIGN OF ABUTMENTS

\[ E_{AE} = \frac{1}{2} g \gamma H^2 (1 - k_v) K_{AE} \times 10^{-9} \]

where the seismic active pressure coefficient \( K_{AE} \) is

\[ K_{AE} = \frac{\cos^2 (\phi - \theta - \beta)}{\cos \theta \cos^2 \beta \cos (\delta + \beta + \theta)} \times \left[ 1 - \frac{\sin (\phi + \delta) \sin (\phi - \theta - i)}{\sqrt{\cos (\delta + \beta + \theta) \cos (i - \beta)}} \right]^2 \]

and where

\( g \) = acceleration of gravity (m/sec.\(^2\))

\( \gamma \) = density of soil (kg/m\(^3\))

\( H \) = height of soil face (mm)

\( \phi \) = angle of friction of soil (\(^\circ\))

\( \theta \) = arc tan \( (k_h / (1 - k_v)) \) (\(^\circ\))

\( \delta \) = angle of friction between soil and abutment (\(^\circ\))

\( k_h \) = horizontal acceleration coefficient (dim.)

\( k_v \) = vertical acceleration coefficient (dim.)

\( i \) = backfill slope angle (\(^\circ\))

\( \beta \) = slope of wall to the vertical, negative as shown (\(^\circ\))
The equivalent expression for passive force if the abutment is being pushed into the backfill is:

\[ E_{pe} = \frac{1}{2} g \gamma H^2 (1 - k_v) K_{pe} \times 10^{-9} \]

where:

\[ K_{pe} = \frac{\cos^2 (\phi - \theta + \beta)}{\cos \theta \cos^2 \beta \cos(\delta - \beta + \theta)} \times \left[ 1 - \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \theta + \delta)}{\cos(\delta - \beta + \theta) \cos(\delta - \beta)}} \right]^2 \]

**How to Estimate Horizontal Earthquake Coefficient?**

- The Seismic Force the wall is subjected to depends upon the deformability of the wall

- If the wall is free to displace at the top, AASHTO suggests the following relationship for estimating EQ Coefficient

\[ k_h = 1.66 A \left( \frac{A}{d} \right)^{0.25} \]  

(C11.6.5-1)

where:

- \( A \) = the maximum earthquake acceleration (dim.)
- \( k_h \) = horizontal seismic acceleration coefficient (dim.)
- \( d \) = the lateral wall displacement (mm)
How to Estimate Horizontal Earthquake Coefficient?

- The Previous formula may be used with confidence in Seismic Zones 1 & 2.
- For Zones 3 & 4 the advice of an earthquake engineering expert may be sought

APPLICATION OF SEISMIC FORCE

- THE KAE and KPE given by Mononobe-Okabe Theory contain the effect of both the Active and Passive Pressures

- It is customary to separate the seismic force from the Total Force as follows:

  \[ K_E = K_{AE} - K_A \]

  Or

  \[ K_E = K_{PE} - K_P \]

- The Static Component of the Earth Pressure is applied at H/3 and the Seismic Component is applied at 0.6 H

LIMITATIONS OF MONONOBE – OKABE THEORY

- Mononobe-Okabe Theory neglects the effect of the self weight of the wall. This should be taken into account by estimating the seismic forces that would be induced in the wall itself and those transferred to the abutment from the superstructure.